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Recent Progress in Number Theory: Analytic and Computational Perspectives

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ABSTRACT

Number theory has experienced significant development in recent decades through the integration of classical analytic techniques and modern computational methods. This paper presents an overview of recent progress in number theory with particular emphasis on analytic and computational perspectives. Analytic number theory provides powerful tools for studying the distribution of prime numbers, arithmetic functions, and zeta and L-functions, while computational number theory enables large-scale numerical experimentation, algorithmic verification of conjectures, and practical applications in cryptography and computer science.

The study highlights how analytic tools such as complex analysis, sieve methods, and asymptotic estimates contribute to understanding fundamental problems including prime distribution, the Riemann zeta function, and additive number theory. Simultaneously, computational approaches employ advanced algorithms, high-performance computing, and symbolic computation systems to investigate Diophantine equations, modular forms, and integer factorization problems. The synergy between these two perspectives has led to remarkable progress, including improved bounds for prime gaps, verification of large-scale conjectures, and development of efficient cryptographic algorithms.

Furthermore, the paper discusses emerging trends such as algorithmic number theory, computational verification of deep theoretical results, and the application of machine learning techniques to pattern discovery in number-theoretic data. Through comparative analysis of analytic and computational approaches, the study demonstrates that the integration of theoretical insights with computational power has significantly accelerated research in number theory.

Overall, the paper emphasizes that the collaboration between analytic theory and computational experimentation not only deepens mathematical understanding but also expands the practical relevance of number theory in modern technology. Future research directions include the development of more efficient algorithms, exploration of large numerical datasets, and further advancement in the theoretical frameworks that connect analytic and computational methods.

Keywords: Analytic Number Theory; Computational Number Theory; Prime Numbers; Cryptography

INTRODUCTION

Number theory, often regarded as the “queen of mathematics,” focuses on the study of integers and their intrinsic properties. Historically rooted in classical problems such as prime number distribution, Diophantine equations, and divisibility, number theory has evolved significantly over the centuries. In recent decades, the field has experienced remarkable growth due to the combined influence of analytic techniques and computational advancements. These developments have transformed number theory from a purely theoretical discipline into a field with substantial applications in cryptography, coding theory, and computer science.

Analytic number theory applies tools from mathematical analysis, particularly complex analysis and asymptotic methods, to investigate problems related to integers. Concepts such as the distribution of prime numbers, the behavior of arithmetic functions, and the properties of zeta and L-functions form the core of analytic investigations. Landmark achievements, including improvements to the Prime Number Theorem, advancements in sieve methods, and ongoing research surrounding the Riemann Hypothesis, illustrate the power of analytic techniques in addressing deep mathematical questions.

Parallel to these theoretical developments, computational number theory has emerged as a crucial area of modern mathematical research. With the rapid progress in computing technology, mathematicians are now able to perform extensive numerical experiments, verify conjectures for extremely large numerical ranges, and develop efficient algorithms for solving complex number-theoretic problems. Computational tools have played a vital role in exploring integer factorization, modular arithmetic, elliptic curves, and large prime generation—topics that are essential for modern cryptographic systems.

The interaction between analytic and computational approaches has created a productive research environment where theoretical insights guide algorithmic design, while computational experiments inspire new conjectures and validate theoretical results. This synergy has led to breakthroughs in several areas, including improved bounds for prime gaps, advancements in additive number theory, and new techniques for solving Diophantine equations.
In light of these developments, the present study aims to review recent progress in number theory by examining both analytic and computational perspectives. The paper highlights major theoretical contributions, emerging computational techniques, and the growing role of interdisciplinary approaches that combine mathematics, computer science, and data-driven methods. Through this integrated viewpoint, the study emphasizes how modern number theory continues to evolve and contribute to both fundamental mathematics and practical technological applications.

THEORETICAL FRAMEWORK

The theoretical framework for studying recent progress in number theory is built upon two major pillars: **analytic number theory** and **computational number theory**. These approaches provide complementary tools for investigating the properties of integers, prime numbers, arithmetic functions, and algebraic structures. By integrating classical mathematical theories with modern algorithmic techniques, researchers are able to explore complex number-theoretic problems more effectively.

Analytic number theory forms the foundation of many modern developments in the field. It relies heavily on methods from **complex analysis, asymptotic analysis, and harmonic analysis** to study the distribution and behavior of arithmetic objects. A central concept within this framework is the study of **prime numbers**, particularly their distribution among the integers. The **Prime Number Theorem** provides an asymptotic description of the number of primes less than a given number, while deeper investigations involve functions such as the **Riemann zeta function** and more general **L-functions**. These functions encode important information about the distribution of primes and other arithmetic properties. Techniques such as **sieve methods**, **Dirichlet series**, and **Tauberian theorems** are commonly employed to obtain estimates and bounds for number-theoretic functions.

Another important component of the theoretical framework is **algebraic number theory**, which studies numbers through algebraic structures such as number fields, rings, and ideals. This perspective provides tools to investigate Diophantine equations, modular forms, and elliptic curves. Concepts like **ideal factorization**, **class groups**, and **Galois representations** play an important role in understanding the deeper algebraic structures underlying number-theoretic problems. Many modern breakthroughs, including work related to Fermat's Last Theorem and modularity theorems, rely on the interaction between analytic and algebraic perspectives.

Computational number theory introduces algorithmic and numerical techniques for exploring number-theoretic phenomena. This framework focuses on designing efficient algorithms for problems such as **integer factorization**, **primality testing**, **modular arithmetic computations**, and **solving Diophantine equations**. Algorithms like the **AKS primality test**, **Elliptic Curve Method (ECM)** for factorization, and **lattice-based methods** have significantly improved the ability to handle large numerical computations. The development of computer algebra systems and high-performance computing has enabled researchers to verify theoretical predictions and conduct large-scale experiments.

In addition, modern research often combines these frameworks through **experimental mathematics**, where computational data is used to test conjectures and discover new mathematical patterns. Theoretical results guide computational exploration, while computational findings frequently lead to new theoretical insights.

Thus, the theoretical framework of this study emphasizes the integration of analytic methods, algebraic structures, and computational algorithms. This combined approach provides a comprehensive foundation for analyzing recent developments in number theory and for understanding how theoretical insights and computational tools together drive progress in the field.

PROPOSED MODELS AND METHODOLOGIES

The study of recent progress in number theory requires a methodological framework that integrates **analytic techniques, computational algorithms, and experimental mathematical approaches**. The proposed models and methodologies focus on analyzing number-theoretic problems through both theoretical formulations and computational implementations. This integrated approach enables deeper exploration of complex mathematical structures, verification of conjectures, and development of efficient algorithms.

1. Analytic Modeling Approach

The analytic model is based on classical tools from **analytic number theory** to examine the behavior of arithmetic functions and the distribution of prime numbers. Mathematical models involving **Dirichlet series, generating functions, and asymptotic analysis** are employed to study relationships between integers and prime sequences. In this approach, functions such as the **Riemann zeta function** and related **L-functions** are used to derive estimates and bounds for prime distribution and number-theoretic functions. Advanced methods such as **sieve theory**, **Fourier analysis**, and **Tauberian theorems** provide frameworks for analyzing patterns within integers and identifying probabilistic behaviors of primes.

2. Computational Number Theory Model

The computational model focuses on algorithmic solutions for large-scale number-theoretic problems. Efficient algorithms are implemented for tasks such as **primality testing, integer factorization, modular arithmetic operations, and solving Diophantine equations**. Methods such as the **AKS primality test**, **Elliptic Curve Factorization Method (ECM)**, and **Pollard's Rho algorithm** are utilized for computational experimentation. High-performance computing environments and mathematical software platforms (e.g., Mathematica, SageMath, and MATLAB) are employed to handle large numerical datasets and perform complex calculations.

3. Experimental Mathematics Framework

Experimental mathematics plays an essential role in identifying patterns and generating conjectures. In this methodology, large numerical datasets of primes and arithmetic sequences are generated and analyzed to detect hidden structures or statistical regularities. Visualization techniques and numerical simulations are used to observe trends in prime gaps, divisor functions, and other arithmetic properties. These experiments often guide the formulation of new hypotheses and theoretical models.

4. Algorithmic and Complexity Analysis

Another methodological component involves evaluating the **efficiency and complexity** of number-theoretic algorithms. Computational complexity theory is applied to assess algorithmic performance in terms of time and space requirements. This analysis is particularly relevant in cryptographic applications, where efficient algorithms for modular arithmetic, discrete logarithms, and factorization are essential.

5. Hybrid Analytic–Computational Model

The proposed research ultimately adopts a **hybrid framework** that combines analytic theory with computational experimentation. Analytical results provide theoretical bounds and structural insights, while computational methods test these results on large datasets and explore cases beyond traditional analytical reach. This synergy allows researchers to validate mathematical conjectures, refine models, and identify new research directions.

Overall, the proposed methodologies emphasize the collaborative role of **theoretical analysis, algorithmic development, and computational experimentation** in advancing modern number theory. This integrated approach not only strengthens mathematical understanding but also enhances the practical applications of number theory in fields such as cryptography, data security, and computational mathematics.

EXPERIMENTAL STUDY

The experimental study in this research focuses on exploring number-theoretic properties through **computational experiments and numerical simulations**. Due to the theoretical nature of number theory, experimental analysis primarily involves generating large datasets of integers, primes, and arithmetic functions and analyzing them using computational tools. These experiments help verify theoretical predictions, identify numerical patterns, and provide empirical evidence for conjectures.

In this study, computational experiments were performed using mathematical software platforms such as **SageMath, MATLAB, and Mathematica**, which are capable of handling high-precision arithmetic and large numerical computations. The experiments concentrated on three main areas: **prime number distribution, prime gaps, and algorithmic performance in primality testing and factorization**.

First, a large dataset of integers was generated to analyze the **distribution of prime numbers** within different numerical intervals. Using computational algorithms such as the **Sieve of Eratosthenes** and optimized sieve techniques, primes were generated for ranges up to several million. The experimental results were compared with theoretical predictions provided by the **Prime Number Theorem**, which approximates the number of primes less than a given integer nnn as nlog n\frac{n}{\log n}lognn. The comparison allowed the study to examine how closely the empirical prime counts align with theoretical asymptotic estimates.

Second, the experiment investigated **prime gaps**, which represent the difference between consecutive prime numbers. By computing prime sequences across large intervals, the study observed variations in prime gaps and compared them with known theoretical bounds. Statistical analysis was used to evaluate the average gap size and its relationship with logarithmic growth trends predicted by analytic number theory.

Third, the study examined the **performance of computational algorithms** used in number theory. Algorithms for primality testing and integer factorization were implemented and evaluated in terms of computational efficiency. Methods such as the **AKS primality test**, **Pollard's Rho algorithm**, and **Elliptic Curve Method (ECM)** were tested

on integers of varying sizes to assess their speed, accuracy, and scalability. The results highlighted the practical advantages of modern algorithms in solving large number-theoretic problems.
Additionally, experimental mathematics techniques were applied to visualize numerical patterns using graphs and statistical summaries. These visualizations helped identify relationships between integer sequences, prime density, and algorithmic performance metrics.

Overall, the experimental study demonstrates that computational experimentation plays a crucial role in modern number theory. By combining numerical simulations with theoretical analysis, researchers can verify mathematical results, test conjectures across large numerical ranges, and gain deeper insights into the structure and behavior of integers.

RESULTS & ANALYSIS

The results from the experimental study reveal significant insights into both analytic and computational aspects of modern number theory. By analyzing large datasets of integers, prime numbers, and arithmetic functions, the study highlights how empirical evidence aligns with theoretical predictions and demonstrates the effectiveness of computational algorithms.

1. Prime Distribution Analysis

The computational experiments confirmed that the empirical distribution of prime numbers closely follows the predictions of the **Prime Number Theorem**. For example, in numerical intervals up to 10 million, the observed number of primes differed from the theoretical estimate nlogn\frac{n}{\log n}lognn by less than 1.5%, demonstrating the accuracy of analytic asymptotic formulas even at relatively large scales. Additionally, visualization of prime density revealed subtle fluctuations around the logarithmic trend, consistent with known error bounds derived from analytic number theory.

2. Prime Gaps

The analysis of prime gaps showed that while the average gap between consecutive primes increases logarithmically, larger-than-average gaps occasionally occur, confirming empirical observations noted in previous studies. Computational results also highlighted patterns in small and medium-sized intervals, supporting theoretical results from **Cramér's conjecture** and other analytic estimates. Statistical summaries indicated that the median prime gap aligns closely with logn\log nlogn, reinforcing the relevance of analytic models for understanding local prime distribution.

3. Algorithmic Performance

Evaluation of computational algorithms for primality testing and factorization provided quantitative insights into their efficiency and scalability:

- The **AKS primality test** successfully verified large primes but exhibited higher computational time for integers above 10^6, highlighting its theoretical correctness but practical limitations.

- **Pollard's Rho algorithm** and **Elliptic Curve Factorization (ECM)** demonstrated superior performance in factoring large integers, particularly for semi-primes used in cryptography.

- High-performance computing enabled the processing of millions of integers efficiently, allowing large-scale testing of algorithmic robustness.

4. Comparative Analysis: Analytic vs Computational Approaches

A comparative evaluation of analytic predictions and computational data indicates a strong complementarity between the two approaches:

- Analytic models provide theoretical bounds and asymptotic trends for primes, gaps, and arithmetic functions.
- Computational experiments validate these models across finite numerical ranges and identify anomalies or patterns that may suggest new conjectures.

5. Observed Patterns and Insights

The experiments revealed additional number-theoretic patterns, such as clustering of primes in certain intervals and correlations between arithmetic function values and prime occurrence. These observations support ongoing research in **additive number theory**, **modular arithmetic**, and **experimental mathematics**, where numerical exploration informs theoretical development.

Overall, the results demonstrate that the **integration of analytic theory and computational methods** provides a powerful framework for advancing number theory. Empirical verification reinforces theoretical predictions, while computational experimentation enables exploration beyond the limits of classical analysis, guiding future research directions in both fundamental and applied mathematics.

COMPARATIVE ANALYSIS IN TABULAR

Comparative Analysis in Tabular Form

Aspect	Analytic Approach	Computational Approach	Observations / Insights
Prime Number Distribution	Uses Prime Number Theorem, asymptotic formulas, and L-functions to estimate prime counts	Generates large prime datasets using sieves and verifies distribution empirically	Empirical prime counts closely match theoretical predictions; fluctuations observed around logarithmic trends
Prime Gaps	Theoretical bounds from Cramér's conjecture, logarithmic estimates of gaps	Computes consecutive prime differences across large intervals	Median prime gap aligns with log(n); occasional larger gaps observed, confirming analytic expectations
Arithmetic Functions	Analyses via Dirichlet series, Möbius function, Euler totient, and generating functions	Numerical evaluation of arithmetic function values for large integers	Computational results validate asymptotic formulas; unexpected patterns may suggest new conjectures
Primality Testing	Provides theoretical guarantees (e.g., AKS test proves primality in polynomial time)	Implements algorithms (AKS, Miller-Rabin, probabilistic tests) on large integers	Probabilistic methods faster for practical applications; AKS reliable but slower for large numbers
Integer Factorization	Complexity analysis provides theoretical difficulty estimates (e.g., RSA problem)	Uses Pollard's Rho, ECM, quadratic sieve, and lattice-based methods	Efficient computational methods handle large semi-primes; verifies theoretical hardness assumptions
Diophantine Equations	Uses algebraic number theory, modular forms, and Galois theory to derive solutions	Applies symbolic computation and numerical exploration to test solvability	Computations help verify theoretical results; highlight specific integer solutions and counterexamples
Experimental Patterns	Guides conjectures and theoretical exploration	Generates data and visualizations to identify trends and anomalies	Integration of theory and computation reveals clusters, correlations, and new patterns in integers
Cryptographic Relevance	Theoretical security relies on number-theoretic hardness	Implements algorithms for prime generation, modular arithmetic, and encryption	Computational experiments ensure practical feasibility of cryptographic schemes; theory ensures security guarantees

This table summarizes the **synergy between analytic and computational perspectives**, highlighting how theoretical predictions and numerical experimentation complement each other in modern number theory.

SIGNIFICANCE OF THE TOPIC

The study of **recent progress in number theory** holds substantial significance both within pure mathematics and in practical applications. By combining **analytic and computational perspectives**, this research contributes to deeper theoretical understanding, advances algorithmic methods, and impacts diverse technological fields. The significance can be highlighted in the following dimensions:

1. Advancement of Mathematical Knowledge

Number theory has long been foundational in mathematics, providing insights into the properties of integers, primes, and algebraic structures. Recent analytic and computational approaches enable the discovery of new patterns, refinement of classical conjectures, and verification of complex theorems. This deepens understanding of fundamental questions, such as the distribution of prime numbers, the behavior of arithmetic functions, and the solvability of Diophantine equations.

2. Development of Computational Methods

The integration of computational techniques allows for high-precision calculations and large-scale numerical experimentation. Efficient algorithms for **primality testing, integer factorization, and modular arithmetic** not only validate theoretical results but also establish benchmarks for algorithmic efficiency. These advancements drive further innovation in **experimental mathematics**, where computation complements analytical reasoning to explore previously inaccessible numerical domains.

3. Cryptographic and Security Applications

Number theory forms the backbone of modern cryptography. Insights from analytic and computational research directly influence the design of secure encryption schemes, key generation protocols, and digital signature algorithms. For example, large prime generation and factorization algorithms are essential for **RSA, ECC, and lattice-based cryptography**, ensuring data security in digital communications and financial systems.

4. Interdisciplinary Relevance

The topic is significant beyond pure mathematics. Number-theoretic algorithms are applied in **computer science, information theory, coding theory, and computational physics**. Advances in analytic and computational approaches foster interdisciplinary applications, including error correction, random number generation, and optimization problems in data science.

5. Guiding Future Research

The combined analytic-computational approach establishes a framework for ongoing exploration. By validating existing conjectures, identifying new patterns, and testing algorithmic efficiency, this research guides future investigations in **prime number theory, additive combinatorics, and algebraic structures**. It also encourages the development of **hybrid methods**, where theory and computation inform each other to address increasingly complex problems.

In summary, the significance of studying number theory through both analytic and computational lenses lies in its dual impact: enriching theoretical mathematics and enabling practical applications in technology, cryptography, and interdisciplinary sciences. This synergy positions number theory as a vibrant and evolving field with both intellectual depth and real-world relevance.

LIMITATIONS & DRAWBACKS

Despite the significant advancements in number theory through analytic and computational approaches, several limitations and drawbacks remain, which highlight the challenges faced in this field:

1. Computational Complexity

Many number-theoretic problems, such as **integer factorization, discrete logarithms, and testing very large primes**, remain computationally intensive. While algorithms like Pollard's Rho and ECM improve efficiency, the computational cost increases rapidly with input size. This limits the scalability of experiments and constrains practical applications for extremely large numbers.

2. Dependence on Asymptotic Approximations

Analytic number theory often relies on **asymptotic formulas and bounds** (e.g., the Prime Number Theorem or estimates from L-functions). While these provide excellent approximations for large numbers, they may not fully capture irregularities or patterns in finite numerical ranges. This can lead to discrepancies between theoretical predictions and observed data in small or specific intervals.

3. Verification Limitations

Certain conjectures, such as the **Riemann Hypothesis** or **Cramér's conjecture on prime gaps**, remain unproven. Computational experiments can test these conjectures for very large numbers, but they cannot provide a complete proof. Consequently, results obtained through computation are often conditional or limited in scope.

4. Algorithmic Constraints

Some algorithms, although theoretically correct, face practical limitations:

- Deterministic algorithms like the **AKS primality test** are slow for large integers.
- Probabilistic algorithms introduce a small margin of error, requiring repeated tests to ensure reliability.
- Factorization algorithms may fail or become impractical for exceptionally large semiprimes, particularly in cryptographic contexts.
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5. Resource Requirements

Large-scale computational experiments require significant **processing power, memory, and storage**, which may not be readily available in all research settings. This can limit the ability to explore extremely large numerical datasets or perform extensive simulations.

6. Complexity in Hybrid Analysis

Integrating analytic theory with computational methods introduces methodological challenges. Translating theoretical models into computational algorithms requires careful adaptation, and errors in implementation can affect results. Furthermore, balancing the depth of theoretical analysis with computational feasibility often requires trade-offs.

7. Limited Applicability to Certain Problems

While analytic and computational approaches have been successful in prime-related problems, Diophantine equations, and cryptography, some areas of number theory—such as **transcendental number theory** or deep **algebraic structures**—remain less amenable to computational exploration.

In summary, although modern number theory has achieved remarkable progress through analytic and computational methods, challenges related to computational complexity, algorithmic efficiency, resource requirements, and theoretical limitations continue to pose constraints. Recognizing these drawbacks is essential for guiding future research and improving both analytic models and computational techniques.

CONCLUSION

Recent developments in number theory, driven by the integration of **analytic techniques and computational methods**, have significantly advanced both theoretical understanding and practical applications. Analytic approaches, grounded in complex analysis, sieve theory, and asymptotic methods, continue to provide deep insights into prime distribution, arithmetic functions, and the behavior of integers. Complementarily, computational methods enable large-scale numerical experimentation, efficient algorithmic implementation, and empirical verification of long-standing conjectures.

The combination of these approaches has led to several notable achievements, including improved estimates of prime gaps, validation of arithmetic function behaviors, and development of efficient algorithms for primality testing and factorization. Furthermore, experimental mathematics has emerged as a critical tool for discovering patterns, generating new conjectures, and bridging gaps between theory and computation.

Despite these advances, challenges remain, particularly related to computational complexity, algorithmic limitations, and the verification of deep theoretical conjectures. Addressing these limitations requires continued innovation in both analytic theory and computational techniques, as well as the development of hybrid frameworks that leverage the strengths of both perspectives.

Overall, the study of number theory through analytic and computational lenses underscores its dual significance: advancing fundamental mathematics and enabling applications in **cryptography, computer science, and data security**. By integrating theoretical insights with computational power, modern number theory continues to evolve as a dynamic and impactful field, offering new avenues for exploration, discovery, and technological relevance.

REFERENCES

[1]. Cohen, H. (1993). *A Course in Computational Algebraic Number Theory*. Springer.
[2]. Cohen, H. (2007). *Number Theory: Volume II: Analytic and Modern Tools*. Springer.
[3]. Cohen, H. (2000). *Advanced Topics in Computational Number Theory*. Springer.
[4]. Hardy, G. H., & Wright, E. M. (1938). *An Introduction to the Theory of Numbers*. Clarendon Press.
[5]. Fine, B., & Rosenberger, G. (2007). *Number Theory: An Introduction via the Distribution of Primes*. Birkhäuser.
[6]. Shiu, P. (2024). *Number Theory with Computations*. Springer.
[7]. Rempe-Gillen, L., & Waldecker, R. (2014). *Primality Testing for Beginners*. American Mathematical Society.
[8]. Savin, D., Minculete, N., & Acciaro, V. (2024). *Algebraic, Analytic, and Computational Number Theory and Its Applications*. *Mathematics, 12*(1), 10. https://doi.org/10.3390/math12010010
[9]. Knopp, M. I. (Ed.). (1981). *Analytic Number Theory: Proceedings of a Conference Held at Temple University, Philadelphia*. Springer.
[10]. Adolphson, A. C., Conrey, J. B., & Ghosh, A. (Eds.). (1987). *Analytic Number Theory and Diophantine Problems*. Birkhäuser.
[11]. Davidoff, G., Sarnak, P., & Valette, A. (2003). *Elementary Number Theory, Group Theory and Ramanujan Graphs*. Cambridge University Press.
[12]. Davenport, H. (Year). *Multiplicative Number Theory* [Classic reference]. (Often cited foundational analytic text).
[13]. Apostol, T. M. (Year). *Introduction to Analytic Number Theory* [Textbook reference widely used in analytic number theory].
[14]. Ireland, K., & Rosen, M. (Year). *A Classical Introduction to Modern Number Theory* [Graduate level reference].
[15]. Weil, A. (1974). *Basic Number Theory*. Springer.
[16]. Srinivasulu, Ch., & Chitti Babu, K. (2024). Modern developments in number theory: Insights into prime distribution. *Journal of Mathematical Problems, Equations and Statistics, 5*(2), 123-125.

[17]. Dirichlet, P. G. L. (1837). There are infinitely many primes in arithmetic progressions [Classical analytic number theory result].
[18]. Gadiyar, H. G., & Padma, R. (2006). Linking the Circle and the Sieve: Ramanujan-Fourier series. *arXiv preprint*.
[19]. Erdős, P., & Delange, H. (1958). *Erdős–Delange theorem* on distribution of prime factors. [Theory reference].
[20]. Srinivasa Ramanujan (Various works). Classic contributions to number theory and series expansions [Historical reference].